Int. J. Heat Mass Transfer. Vol. 31, No. 4, pp. 855–861, 1988
Printed in Great Britain

0017–9310/88 \$3.00+0.00
Pergamon Press plc

Drying behaviour of spherical grains

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(*Received* 20 *November* 1986 *and in final form* 25 *May* 1987)

Abstract—The Crank–Nicholson implicit numerical method has been applied to solve the diffusion equation with variable mass diffusivity for spherically-shaped grain. The numerical data so obtained has been presented in the form of a series of curves which represent drying characteristics of a spherical grain in general. The experimental results of drying of grain are correlated with the theoretical results. A correlation describing the diffusivity of the grain with moisture content and temperature is established. When the expression for diffusivity was used for predicting the drying behaviour of the grain, for a known value of initial moisture content, equilibrium moisture and equivalent radius, a good agreement is obtained between the experimental data and the theoretical prediction.

1. INTRODUCTION

THE STUDY of drying behaviour of different materials has been a subject of interest for various investigators during the past 60 years. Drying is a complex thermophysical and physiochemical process involving heat and mass transfer between the surface of the substance and the surrounding medium, and also within the substance. Several mechanisms to explain the phenomenon of migration of moisture in biological porous products have been critically discussed in the review paper by Sharaf-Eldeen *et al.* [1] and the choice of a suitable mechanism for a particular process is dependent on the type of material, moisture bonding and moisture content.

Luikov [2] suggested a comprehensive mathematical model, for describing the drying of capillary porous products, which includes the phenomenological coefficients and the coupling coefficients due to the combined effect of the moisture, temperature and total pressure gradients. It was reported by Brooker *et al.* [3] that the moisture flow due to the pressure gradient is insignificant at temperatures normally used for artificial drying of cereal grains. It is also pointed out by Sharaf-Eldeen *et al.* [1] and Brooker *et al.* [3] that the temperature gradient need not be considered in most of the grain-drying applications where the thermal diffusivity is large compared to the moisture diffusivity. In view of this, use of the diffusion equation is justified in describing the moisture movement in fully-exposed drying of cereal grains in the falling rate period. The diffusion equation is expressed as

$$\frac{\partial M}{\partial t} = \nabla \cdot (D \cdot \nabla M). \tag{1}$$

In the early period of study, Sherwood [4–6] and Newman [7] used equation (1) to correlate drying data of fully exposed porous solids by assuming the diffusivity coefficient to be constant. Later on, Becker [8], Hustrulid and Flikke [9] and Hamdy and Barre [10] obtained their solutions under the same assumption.

However, Hamdy and Johnson [11] found, during their drying experiment on hay wafers, that diffusivity varies with moisture content of the substance. But investigations by Henderson and Pabis [12] on maize, Wang and Singh [13] and Steffe and Singh [14] on rice revealed that the diffusion coefficient is a function of drying temperature. On the other hand, results obtained by Chu and Hustrulid [15] and Husain *et al.* [16] established a correlation for diffusivities of corn and rice, respectively, as a function of temperature and moisture content.

It is inferred from the foregoing review that the diffusivity varies with both moisture and temperature of grain during a drying process; however, very few studies have been conducted to support this point of view. Therefore, there is a need to investigate on this aspect further and to establish an approach which can be used for predicting the drying behaviour of a grain under specified working conditions. It is with this view that the present work has been taken up.

2. EXPERIMENTAL SET-UP AND PROCEDURE

The experimental drying apparatus was designed and fabricated [17] for carrying out a single-layer drying of a grain as shown in Fig. 1. It consists of three units; a humidification chamber, an air heating system, and a drying unit. The drying unit accommodated five wire-mesh-bottomed trays for placing grain samples in them for the purpose of drying. The orificemeter used in the set-up was designed as per ISO specifications.

Prior to conducting the experiment, the equipment was run for about an hour to achieve the steady state and desired temperature and humidity of the drying

NOMENCLATURE

a, b	coefficients, equation (7)
B	diffusivity parameter
D	diffusivity [$m^2 h^{-1}$]
$\bar{D}$	non-dimensional diffusivity, D/D_e
DBT	dry-bulb temperature [K]
DPT	dew-point temperature [K]
M	moisture content, percent dry basis
$\bar{M}$	moisture ratio, $(M-M_e)/(M_0-M_e)$
N	number of nodal points in radial direction
R	equivalent radius of grain [m]
r	radial coordinate [m]
$\bar{r}$	non-dimensional radius, r/R
$\Delta\bar{r}$	non-dimensional distance between two consecutive nodes in the radial direction
T	drying air temperature [K]
t	time [h].

Greek symbols

λ	$\bar{D}_j\Delta\tau/(\Delta\bar{r})^2$
ϕ	$b(M_0-M_e)/8$
τ	non-dimensional time, $D_e t/R^2$
$\Delta\tau$	non-dimensional time step.

Superscript

$+$	value corresponding to $(\tau+\Delta\tau)$.

Subscripts

0	value at $t = 0$
av	average taken over the whole grain
e	equilibrium state
j	nodal position in the radial direction.

air. A sample of the rewetted gram (*Cicer arietinum* L., sphericity 82%) weighing about 110 g was placed in each tray and the drying rate of the material was observed under a set of desired drying conditions. During the test, observations were made for the following quantities: (a) dry- and wet-bulb temperatures of the inlet air, (b) wet-bulb temperature of the air at the exit of the humidification chamber, (c) dry- and wet-bulb temperatures of the air in the drying unit, (d) mass of the sample at regular intervals of time, (e) pressure drop across the orificemeter and the barometric pressure. Such data was recorded until the difference between two consecutive readings of a drying sample was negligible. The experimental observations were made under the following conditions:

(a) initial moisture content: about 35% dry basis;
(b) average air velocity of drying air: 0.4 m s^{-1};
(c) dry-bulb temperature of the drying air: 323, 328, 333, 343, 348, 353, 358 and 363 K with two different dew-point temperatures (approximately 290 and 303 K).

3. METHOD OF SOLUTION

The diffusion equation representing the isothermal drying process can be expressed in spherical coordinates as

$$\frac{\partial M}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D \frac{\partial M}{\partial r}\right]. \tag{2}$$

It is to be noted that the pressure and temperature gradients have negligible effect on the moisture movement in drying of most of the cereal grains [1, 3]. The above equation is subjected to the following initial and boundary conditions:

$$M = M_0 \quad \text{at } t = 0 \quad \text{and} \quad 0 \leqslant r \leqslant R \tag{3a}$$

$$M = M_e \quad \text{at } t > 0 \quad \text{and} \quad r = R \tag{3b}$$

$$\frac{\partial M}{\partial r} = 0 \quad \text{at } t > 0 \quad \text{and} \quad r = 0. \tag{3c}$$

It is assumed that at $r = R$, the surface resistance to vapour diffusion is negligible, implying thereby that

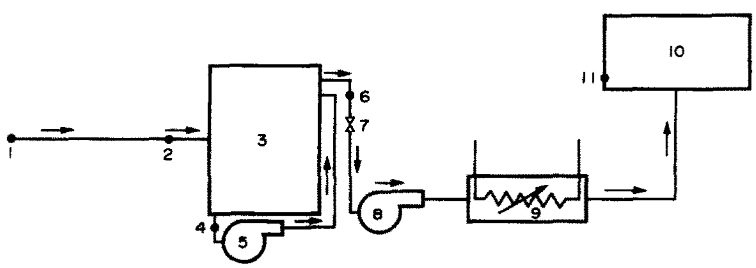

FIG. 1. Schematic diagram of the experimental set-up: 1, thermometers to measure dry- and wet-bulb temperatures of the inlet air; 2, orifice-meter; 3, humidification chamber; 4, thermometer to measure water temperature; 5, water pump; 6, thermometers to measure wet-bulb temperature of humidified air; 7, gate valve; 8, blower; 9, air heating system; 10, drying unit; 11, thermometers to measure drying air dry- and wet-bulb temperatures.

the liquid concentration on the grain surface falls to the equilibrium value immediately at the start of the falling rate period of the drying [4]. Introducing dimensionless variables

$$\bar{M} = \frac{M - M_e}{M_0 - M_e}, \quad \bar{r} = \frac{r}{R}, \quad \bar{D} = \frac{D}{D_e} \quad \text{and} \quad \tau = \frac{D_e t}{R^2} \tag{4}$$

equation (2) transforms into a non-linear partial differential equation in the form

$$\frac{\partial \bar{M}}{\partial \tau} = \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left[\bar{r}^2 \bar{D} \frac{\partial \bar{M}}{\partial \bar{r}} \right]. \tag{5}$$

The initial and boundary conditions then become

$$\bar{M} = 1 \quad \text{at} \quad \tau = 0 \quad \text{and} \quad 0 \leqslant \bar{r} \leqslant 1 \tag{6a}$$

$$\bar{M} = 0 \quad \text{at} \quad \tau > 0 \quad \text{and} \quad \bar{r} = 1 \tag{6b}$$

$$\frac{\partial \bar{M}}{\partial \bar{r}} = 0 \quad \text{at} \quad \tau > 0 \quad \text{and} \quad \bar{r} = 0. \tag{6c}$$

Assuming

$$D = a \exp(bM) \tag{7}$$

which leads to

$$D_e = a \exp(bM_e) \tag{8}$$

and

$$\bar{D} = \exp[b(M_0 - M_e)\bar{M}] \tag{9}$$

where a and b are assumed to be functions of the drying temperature.

A combination of equations (5) and (9) yields

$$\frac{\partial \bar{M}}{\partial \tau} = \bar{D} \left[\frac{2}{\bar{r}} \frac{\partial \bar{M}}{\partial \bar{r}} + b(M_0 - M_e) \left(\frac{\partial \bar{M}}{\partial \bar{r}} \right)^2 + \frac{\partial^2 \bar{M}}{\partial \bar{r}^2} \right]. \tag{10}$$

It is not possible to obtain a closed-form solution of equation (10) and therefore a numerical method is resorted to. For this purpose, equi-spaced nodal points are taken in the $\bar{r}$-direction and the Crank–Nicholson implicit method is used to transform the partial differential equation (10) at a point $(\bar{r}, \tau + \Delta\tau/2)$ under the following simplifying assumption for obtaining fast convergence:

$$\left(\frac{\partial \bar{M}}{\partial \bar{r}} \right)^2_{\bar{r}, \tau + \Delta\tau/2} = \left(\frac{\partial \bar{M}}{\partial \bar{r}} \right)_{\bar{r}, \tau} \cdot \left(\frac{\partial \bar{M}}{\partial \bar{r}} \right)_{\bar{r}, \tau + \Delta\tau/2}. \tag{11}$$

Upon this, the resulting finite-difference equation is obtained in the following form:

$$\left[- \left\{ \frac{\Delta \bar{r}}{2\bar{r}} + \phi(\bar{M}_{j+1} - \bar{M}_{j-1}) + \tfrac{1}{2} \right\} \right] \bar{M}^+_{j+1} + \left(\frac{\lambda_j + 1}{\lambda_j} \right) \bar{M}^+_j + \left[\frac{\Delta \bar{r}}{2\bar{r}} + \phi(\bar{M}_{j+1} - \bar{M}_{j-1}) - \tfrac{1}{2} \right] \bar{M}^+_{j-1}$$

$$= \left[\left(\frac{\Delta \bar{r}}{2\bar{r}} + \tfrac{1}{2} \right) \bar{M}_{j+1} - \left(\frac{\Delta \bar{r}}{2\bar{r}} - \tfrac{1}{2} \right) \bar{M}_{j-1} + \phi(\bar{M}_{j+1} - \bar{M}_{j-1})^2 - \left(\frac{\lambda_j - 1}{\lambda_j} \right) \bar{M}_j \right] \quad \text{for } 2 \leqslant j \leqslant N \tag{12}$$

where

$$\phi = \frac{b(M_0 - M_e)}{8} \quad \text{and} \quad \lambda_j = \frac{\bar{D}_j \Delta\tau}{(\Delta \bar{r})^2}. \tag{13}$$

An additional finite-difference equation is obtained pertaining to the centre of the grain which is expressed as

$$\bar{M}^+_1 = \frac{3\lambda_1(\bar{M}^+_2 + \bar{M}_2 - \bar{M}_1) + \bar{M}_1}{1 + 3\lambda_1}. \tag{14}$$

Equations (12) and (14) represent a set of simultaneous algebraic equations in the form of a tri-diagonal matrix; these were solved by the Gaussian elimination procedure. During the process of computation, the value of $\Delta\bar{r}$ was taken equal to 0.1, whereas $\Delta\tau$ was taken as 0.001, 0.002, 0.005, 0.01 and 0.05 in the ranges 0–0.01, 0.01–0.02, 0.02–0.04, 0.04–0.10 and 0.10–0.35, respectively. The computation was performed in the following steps for a known value of initial moisture distribution at a particular drying temperature.

(1) The value of the diffusivity parameter B defined by

$$B = b(M_0 - M_e) \tag{15}$$

is initially taken equal to zero.

(2) With the known values of $\bar{M}$ at time τ, at all nodal points in the $\bar{r}$-direction, values of $\bar{M}$ at $\tau + \Delta\tau$ at the corresponding nodes were obtained. Subsequently, the average value of $\bar{M}$ for the grain, $\bar{M}_{av}$, was computed by using the trapezoidal rule of numerical integration. In this way, computations were carried out up to $\tau = 0.35$.

(3) The above steps were repeated for other values of the parameter B up to 6 in steps of 1.

A plot of the theoretical data so generated for the variation of $\bar{M}_{av}$ with τ for different values of B is shown in Fig. 2. To make use of these curves for predicting drying behaviour of grains, it is essential to know M_0, M_e, D_e and R of the grain pertaining to the actual drying conditions. Whereas M_0 and R can easily be measured for a grain, values of M_e and D_e can only be determined from the experimental drying data.

An accepted procedure suggested by Henderson and Pabis [12] has been used for evaluation of M_e. For one of the drying conditions, data for $\bar{M}_{av}$ vs t is shown plotted using the finally obtained M_e in Fig. 3. It is seen that the major portion of the plot is a straight line beyond an initial period of 2 h which satisfies the requirement for obtaining the proper value of M_e according to the aforesaid method.

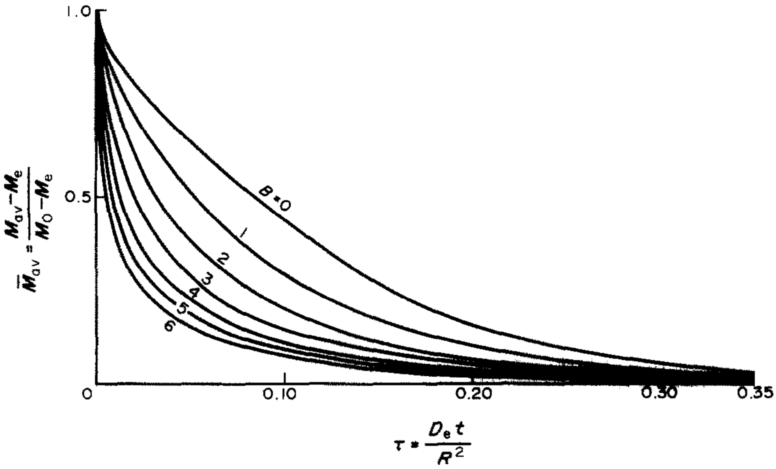

FIG. 2. Variation of moisture ratio with time for different values of diffusivity parameter B.

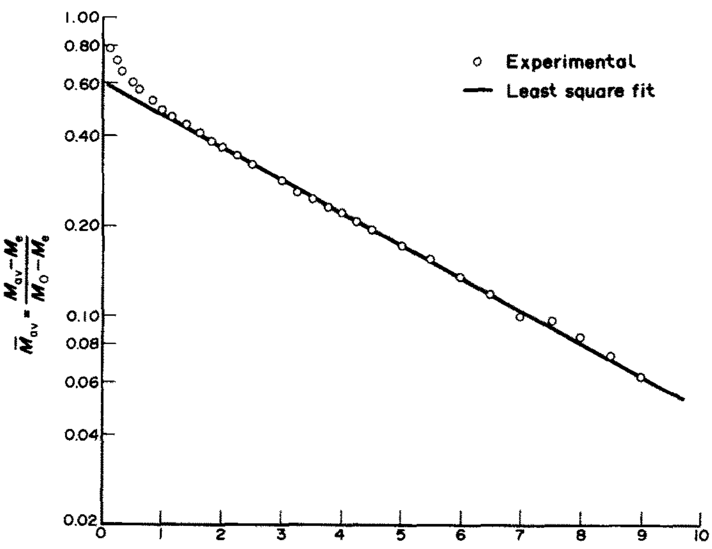

FIG. 3. Determination of M_e at DBT = 338 K and DPT = 289 K.

4. EVALUATION OF THE DRYING PARAMETER *B* AND COEFFICIENTS *a* AND *b*

Out of the four quantities M_0, M_e, D_e and R, D_e remains to be specified in order to transfer the experimental data on the $\bar{M}_{av}$–τ plot, and to choose a suitable value of B so that a reasonable agreement is achieved between the experimental and theoretical drying data. Since the experimental data M_{av} vs t is already known, the corresponding values of $\bar{M}_{av}$ are computed by using

$$\bar{M}_{av} = \frac{M_{av}-M_e}{M_0-M_e}. \tag{16}$$

For the values of $\bar{M}_{av}$ thus obtained, the corresponding values of τ are read off from Fig. 2 for each value of B. By using equation (4), values of D_e are obtained corresponding to each value of τ, thus $\bar{M}_{av}$. The values of D_e for each temperature are plotted against $\bar{M}_{av}$. Such a plot for one of the temperatures is shown in Fig. 4. It is observed from the figure that there is a least variation in the value of D_e in the vicinity of $\bar{M}_{av} = 0.5$ for $B = 5$. Similar plots drawn for other temperatures in the range $323\,\text{K} \leqslant T \leqslant 353\,\text{K}$ revealed that the standard deviation in the value of D_e is least at $B = 5$ in all cases for the grain under consideration.

The values of D_e in the neighbourhood of $\bar{M}_{av} = 0.5$ are most suitable in this analysis as deviation in D_e is within $\pm 10\%$ for $0.4 \leqslant \bar{M}_{av} \leqslant 0.6$. It may be pointed out that the values of $\bar{M}_{av}$ above 0.5, which correspond to the early period of drying are not advisable to be used because of the existence of temperature gradients within the kernel [15]. On the other hand, if the lower values of $\bar{M}_{av}$ are used instead of 0.5, it is found that D_e increases by 17–64% of the value at $\bar{M}_{av} = 0.5$ as $\bar{M}_{av}$ decreases from 0.3 to 0.2.

In order to determine coefficients a and b at each drying temperature for $B = 5$, equation (8) is used in conjunction with equation (15). The following

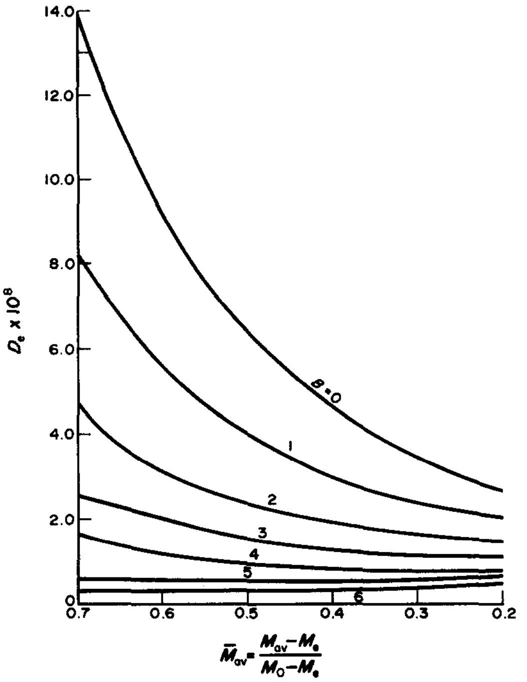

FIG. 4. Variation of equilibrium mass diffusivity with moisture ratio for different values of B at DBT = 323 K and DPT = 287.5 K.

relationships are obtained when the least-square method was used to get a and b as a function of drying temperature:

$$a = 5.79 \times 10^{-16} \exp(0.053T) \tag{17}$$

$$b = 0.916 \exp(-4.947 \times 10^{-3} T). \tag{18}$$

The variation of coefficients a and b with drying air is found to be in agreement with the other investigations [15, 16].

5. DISCUSSION OF RESULTS

The experimental drying data for grain corresponding to two different temperatures 323 and 348 K are shown in Figs. 5 and 6 which depict the variation of moisture ratio with time. It is observed that the drying characteristics of the grain are similar to those of shelled maize, rice, corn, pigeon pea and shelled corn reported by various authors [12, 13, 15, 17, 18]. The theoretical prediction curves corresponding to two different temperatures using variable diffusivity are also drawn for comparison with the experimental results. It is observed from the plots shown in Figs. 5 and 6 that there is a good agreement between the experimental and theoretical results. However, the theory slightly overpredicts the results in the later part of the curve. The start of deviation begins earlier as the drying temperature increases. It may be pointed out that Chu and Hustrulid [15] had to resort to a pseudo initial moisture content to seek agreement between their theoretical and experimental results on corn. They used an explicit central-difference scheme, for the purpose of finite differencing, which needs the stability criterion to be satisfied while choosing a forward time-step during numerical solution of the diffusion equation. However, the method used in the present work is unconditionally stable.

6. CONCLUSIONS

The present theoretical and experimental study related to drying of grains reveals the following points.

(1) The Crank–Nicholson method proves to be a useful tool in the solution of the diffusion equation for generating pertinent drying data.

(2) The drying characteristics of the grain are similar to those of other grains belonging to the same category in the falling rate period.

(3) The relation expressing dependence of diffusivity on moisture and temperature can be used for predicting the drying characteristic of the grain provided the initial moisture content, equilibrium moisture and equivalent radius of the grain at the desired drying temperature are known.

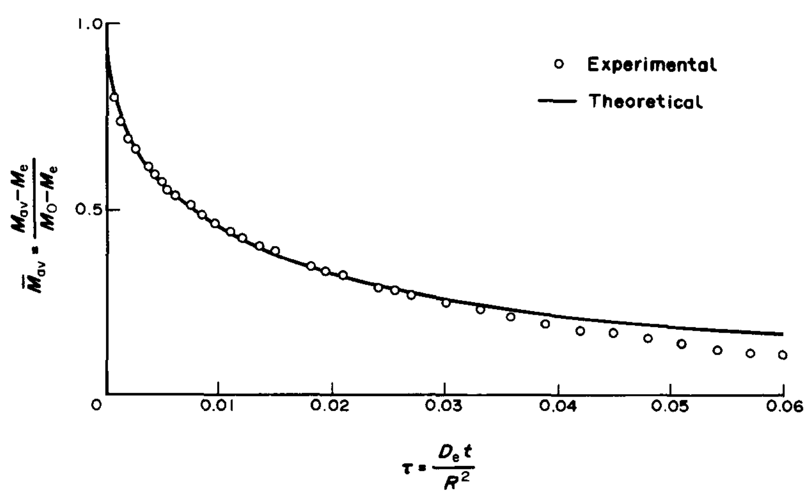

FIG. 5. Variation of moisture ratio with time at DBT = 323 K and DPT = 287.5 K.

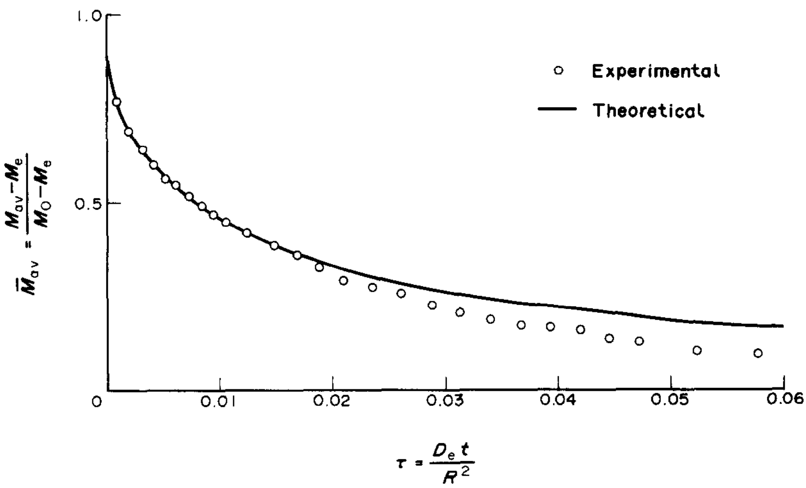

FIG. 6. Variation of moisture ratio with time at DBT = 348 K and DPT = 294.6 K.

(4) The general approach adopted for seeking solution to the drying problem related to grains has generated a versatile numerical data which can be used as such for predicting the drying characteristic of any nearly spherical-shaped grain.

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SECHAGE DES GRAINS SPHERIQUES

Résumé—La méthode numérique implicite de Crank–Nicholson est appliquée à la résolution de l'équation de diffusion avec diffusivité massique variable, pour un grain de forme sphérique. Les résultats numériques sont présentés sous forme d'une série de courbes qui représentent les caractéristiques de séchage d'un grain sphérique en général. Les résultats expérimentaux sont comparés aux résultats théoriques. On établit une formule décrivant la diffusivité du grain avec l'humidité et la température. Quand on l'utilise pour prédire le comportement du séchage du grain, avec une valeur initiale donnée du contenu d'humidité, une humidité d'équilibre et un rayon équivalent, on obtient un bon accord entre les résultats expérimentaux et les prévisions théoriques.

TROCKNUNGSVERHALTEN VON KUGELFÖRMIGEM GETREIDE

Zusammenfassung—Die implizite numerische Methode von Crank–Nicholson wurde verwendet, um die Diffusionsgleichung mit veränderlichem Diffusionskoeffizienten für kugelförmiges Getreide zu lösen. Die Ergebnisse der numerischen Berechnungen wurden als Kurvenschar dargestellt, welche die Trocknungscharakteristik von kugelförmigem Getreide allgemein wiedergeben. Experimentelle Ergebnisse der Getreidetrocknung werden mit den theoretischen verglichen. Eine Gleichung für den Diffusionskoeffizienten als Funktion von Feuchte und Temperatur wird angegeben. Die Verwendung dieser Beziehung für den Diffusionskoeffizienten zur Beschreibung des Trocknungsverhaltens von Getreide führt zu guter Übereinstimmung zwischen theoretischen und experimentellen Werten, wenn die Anfangsfeuchte, das Feuchtegleichgewicht und ein äquivalenter Radius bekannt sind.

ЗАКОНОМЕРНОСТИ СУШКИ СФЕРИЧЕСКИХ ЧАСТИЦ

Аннотация—Для решения нелинейного уравнения диффузии используется неявная разностная схема Кранка–Никольсона. Численные результаты представлены в виде номограмм, которые являются характеристиками сушки сферической частицы. Экпериментальные результаты по сушке частицы согласуются с теоретическими данными. Устанавливается соотношение, описывающее зависимость коэффициента диффузии от влагосодержания и температуры. При использовании данного соотношения в расчетах процесса сушки частицы получено хорошее соответстие между экспериментальными и теоретическими результатами для известного начального влагосодержания, конечного равновесного влагосодержания и эквивалентного радичса.